

Name:

Student No:

$$\Rightarrow abab = ba^2b^2$$

7. Let $r(\theta) = \sin^2\left(\frac{\theta}{2}\right)$. Find the arclength of this curve.

$$aba = ba^2b \Rightarrow a^2ba = aba^2b \quad \text{Toraba:}$$

$$a^2ba = a^2b^{2n}a = (a^2ba)^{2n} = aba^2b \cdot aba^2b \cdot \dots \cdot aba^2b \rightarrow 2n \text{ times}$$

$$= aba \cdot \underbrace{ab \cdot ab \cdot a^2b}_{\text{most}} \cdot \dots \cdot ab \cdot a^2b \rightarrow 2n \text{ times} = \underbrace{aba \cdot ba^2b \cdot a^2b}_{\text{most}} \cdot \dots$$

$$\rightarrow n \text{ times} = ba^2b^2a^2b^2a^2b \cdot \dots \rightarrow n \text{ times} =$$

$$= b \cdot \left[a^2b^2a^2b^2a^2b^2 \cdot \dots \cdot a^2b^2a^2b^2a^2b^2 \right] \cdot b^{-1} \Rightarrow$$

$\rightarrow n \text{ times}$

$$a^2bab = b (a^2b^2a^2b^2a^2b^2)^n = b (a^2b(ba^2b)^2 \cdot b)^n = b (a^2b \cdot aba \cdot aba \cdot b)^n =$$

$$= b (a^2ba \cdot \underbrace{ba^2bab})^n = b (a^2ba \cdot aba \cdot ab)^n = b (a^2 \cdot \underbrace{ba^2b \cdot a^2b})^n =$$

$$= b (a^2 \cdot ab \cdot a \cdot a^2b)^n = b \cdot b^{2n} = b^2$$

$$\Rightarrow b = a^2ba \Rightarrow ab = ba \Rightarrow \text{commute} \Rightarrow$$

$$aba = a^2b = ba^2b = a^2b^2 \Rightarrow b = b^2 \Rightarrow b = \underline{1}$$