Метод на Гаус-Жордан

Зад1.

$$A^{*}x=b \to \begin{pmatrix} 1 & 2 & 0 & 2 \\ 3 & 5 & 5.6 & -3.45 \\ 2 & 7.56 & -2.34 & 2 \\ 0 & -0.89 & 0 & 3.14 \end{pmatrix} {}^{*}x= \begin{pmatrix} -6 \\ 8.89 \\ -4 \\ 5 \end{pmatrix}$$

Решаваме по метода на Гаус-Жордан

(Debug) In[15]:=

$$P = \begin{pmatrix} 1 & 2 & 0 & 2 & -6 \\ 3 & 5 & 5.6 & -3.45 & 8.89 \\ 2 & 7.56 & -2.34 & 2 & -4 \\ 0 & -0.89 & 0 & 3.14 & 5 \end{pmatrix}$$

(Debug) Out[15]=

```
{{1, 2, 0, 2, -6}, {3, 5, 5.6, -3.45, 8.89},
 {2, 7.56, -2.34, 2, -4}, {0, -0.89, 0, 3.14, 5}}
```

Чрез програмен алгоритъм

(Debug) In[72]:=

$$P = \begin{pmatrix} 1 & 2 & 0 & 2 & -6 \\ 3 & 5 & 5.6 & -3.45 & 8.89 \\ 2 & 7.56 & -2.34 & 2 & -4 \\ 0 & -0.89 & 0 & 3.14 & 5 \end{pmatrix}$$

```
n = Length[P];
For[
 (*цикъл по стъпките*)
 m = 1, m ≤ n, m++,
 P[[m]] = P[[m]] / P[[m, m]]; (*първи етап*)
 For[s = 1, s ≤ n, s++,
  If[s ≠ m, P[[s]] = P[[m]] * (-P[[s, m]]) + P[[s]]]
 ]; (*втори етап*)
 Print[P // MatrixForm]
]
```

(Debug) Out[72]=

```
{{1, 2, 0, 2, -6}, {3, 5, 5.6, -3.45, 8.89},
 {2, 7.56, -2.34, 2, -4}, {0, -0.89, 0, 3.14, 5}}
```

$$\begin{pmatrix} 1 & 2 & 0 & 2 & -6 \\ 0 & -1 & 5.6 & -9.45 & 26.89 \\ 0 & 3.56 & -2.34 & -2 & 8 \\ 0 & -0.89 & 0 & 3.14 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 11.2 & -16.9 & 47.78 \\ 0 & 1 & -5.6 & 9.45 & -26.89 \\ 0. & 0. & 17.596 & -35.642 & 103.728 \\ 0. & 0. & -4.984 & 11.5505 & -18.9321 \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & 5.78643 & -18.244 \\ 0. & 1. & 0. & -1.89321 & 6.12199 \\ 0. & 0. & 1. & -2.02557 & 5.895 \\ 0. & 0. & 0. & 1.45504 & 10.4486 \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & 0. & -59.7961 \\ 0. & 1. & 0. & 0. & 19.7171 \\ 0. & 0. & 1. & 0. & 20.4406 \\ 0. & 0. & 0. & 1. & 7.18096 \end{pmatrix}$$

Извод: Намираме че X= $\begin{pmatrix} -59.7961 \\ 19.7171 \\ 20.4406 \\ 7.18096 \end{pmatrix}$

Намиране на детерминантата на матрицата A

(Debug) In[75]:=

```
A = (1    2     0     2
     3    5     5.6  -3.45
     2  7.56  -2.34   2
     0 -0.89    0    3.14);
n = Length[A];
det = 1;
For[
 (*цикъл по стъпките*)
 m = 1, m ≤ n, m++,
 det = det * A[[m, m]];
 A[[m]] = A[[m]] / A[[m, m]]; (*първи етап*)
 For[s = 1, s ≤ n, s++,
  If[s ≠ m, A[[s]] = A[[m]] * (-A[[s, m]]) + A[[s]]]
 ]; (*втори етап*)
 Print[A // MatrixForm]
]
Print["Det = ", det]
```

$$\begin{pmatrix} 1 & 2 & 0 & 2 \\ 0 & -1 & 5.6 & -9.45 \\ 0 & 3.56 & -2.34 & -2 \\ 0 & -0.89 & 0 & 3.14 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 11.2 & -16.9 \\ 0 & 1 & -5.6 & 9.45 \\ 0. & 0. & 17.596 & -35.642 \\ 0. & 0. & -4.984 & 11.5505 \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & 5.78643 \\ 0. & 1. & 0. & -1.89321 \\ 0. & 0. & 1. & -2.02557 \\ 0. & 0. & 0. & 1.45504 \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & 0. \\ 0. & 1. & 0. & 0. \\ 0. & 0. & 1. & 0. \\ 0. & 0. & 0. & 1. \end{pmatrix}$$

Det = -25.6029

Намиране на обратната матрица A

(Debug) In[81]:=

$$AA = \begin{pmatrix} 1 & 2 & 0 & 2 \\ 3 & 5 & 5.6 & -3.45 \\ 2 & 7.56 & -2.34 & 2 \\ 0 & -0.89 & 0 & 3.14 \end{pmatrix}; \; b = \begin{pmatrix} -6 \\ 8.89 \\ -4 \\ 5 \end{pmatrix};$$

(Debug) In[82]:=

$$A = \begin{pmatrix} 1 & 2 & 0 & 2 & 1 & 0 & 0 & 0 \\ 3 & 5 & 5.6 & -3.45 & 0 & 1 & 0 & 0 \\ 2 & 7.56 & -2.34 & 2 & 0 & 0 & 1 & 0 \\ 0 & -0.89 & 0 & 3.14 & 0 & 0 & 0 & 1 \end{pmatrix};$$

```
n = Length[A];
det = 1;
For[
 (*цикъл по стъпките*)
 m = 1, m ≤ n, m++,
 det *= A[[m, m]];
 A[[m]] = A[[m]] / A[[m, m]]; (*първи етап*)
 For[s = 1, s ≤ n, s++,
  If[s ≠ m, A[[s]] += A[[m]] * (-A[[s, m]])]
 ]; (*втори етап*)
 Print[A // MatrixForm]
]
Print["Det = ", det]
Print["Пресметнато с WM det = ", Det[AA]]
Print["Пресметнато с WM solution = ", LinearSolve[AA, b] // MatrixForm]
Print["Пресметнато с WM inverse matrix = ", Inverse[AA] // MatrixForm]
```

$$\begin{pmatrix} 1 & 2 & 0 & 2 & 1 & 0 & 0 & 0 \\ 0 & -1 & 5.6 & -9.45 & -3 & 1 & 0 & 0 \\ 0 & 3.56 & -2.34 & -2 & -2 & 0 & 1 & 0 \\ 0 & -0.89 & 0 & 3.14 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 & 11.2 & -16.9 & -5 & 2 & 0 & 0 \\ 0 & 1 & -5.6 & 9.45 & 3 & -1 & 0 & 0 \\ 0. & 0. & 17.596 & -35.642 & -12.68 & 3.56 & 1. & 0. \\ 0. & 0. & -4.984 & 11.5505 & 2.67 & -0.89 & 0. & 1. \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & 5.78643 & 3.07093 & -0.26597 & -0.636508 & 0. \\ 0. & 1. & 0. & -1.89321 & -1.03546 & 0.132985 & 0.318254 & 0. \\ 0. & 0. & 1. & -2.02557 & -0.720618 & 0.202319 & 0.0568311 & 0. \\ 0. & 0. & 0. & 1.45504 & -0.921562 & 0.118356 & 0.283246 & 1. \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & 0. & 6.73581 & -0.736652 & -1.76293 & -3.97682 \\ 0. & 1. & 0. & 0. & -2.23455 & 0.286983 & 0.686798 & 1.30114 \\ 0. & 0. & 1. & 0. & -2.00353 & 0.367084 & 0.451141 & 1.39211 \\ 0. & 0. & 0. & 1. & -0.633359 & 0.0813424 & 0.194666 & 0.687267 \end{pmatrix}$$

Det = -25.6029

Пресметнато с WM det = -25.6029

Пресметнато с WM solution = $\begin{pmatrix} -59.7961 \\ 19.7171 \\ 20.4406 \\ 7.18096 \end{pmatrix}$

Пресметнато с WM inverse matrix = $\begin{pmatrix} 6.73581 & -0.736652 & -1.76293 & -3.97682 \\ -2.23455 & 0.286983 & 0.686798 & 1.30114 \\ -2.00353 & 0.367084 & 0.451141 & 1.39211 \\ -0.633359 & 0.0813424 & 0.194666 & 0.687267 \end{pmatrix}$

Зад2.Дадена е следната система линейни алгебрични уравнения:

x1 + 2x2 − x4 = 0

−3.12x1 + 5.76x2 − 21x3 = −0.9

89x1 + 7.87x3 = 90

−9.8x2 + 34x4 = −0.34

1. Да се реши по метода на Гаус-Жордан.

(Debug) In[91]:=

$$\mathbf{Z} = \begin{pmatrix} 1 & 2 & 0 & -4 & 0 \\ -3.12 & 5.76 & -21 & 0 & -0.9 \\ 89 & 0 & 7.87 & 0 & 90 \\ 0 & -9.8 & 0 & 34 & -0.34 \end{pmatrix}$$

(Debug) Out[91]=

```
{{1, 2, 0, -4, 0}, {-3.12, 5.76, -21, 0, -0.9},
 {89, 0, 7.87, 0, 90}, {0, -9.8, 0, 34, -0.34}}
```

(Debug) In[92]:=

```
n = Length[Z];
For[
 (*цикъл по стъпките*)
 m = 1, m ≤ n, m++,
 Z[[m]] = Z[[m]] / Z[[m, m]]; (*първи етап*)
 For[s = 1, s ≤ n, s++,
  If[s ≠ m, Z[[s]] = Z[[m]] * (-Z[[s, m]]) + Z[[s]]]
 ]; (*втори етап*)
 Print[Z // MatrixForm]
]
```

$$\begin{pmatrix} 1 & 2 & 0 & -4 & 0 \\ 0. & 12. & -21. & -12.48 & -0.9 \\ 0 & -178 & 7.87 & 356 & 90 \\ 0 & -9.8 & 0 & 34 & -0.34 \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 3.5 & -1.92 & 0.15 \\ 0. & 1. & -1.75 & -1.04 & -0.075 \\ 0. & 0. & -303.63 & 170.88 & 76.65 \\ 0. & 0. & -17.15 & 23.808 & -1.075 \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & 0.0497658 & 1.03356 \\ 0. & 1. & 0. & -2.02488 & -0.516779 \\ 0. & 0. & 1. & -0.56279 & -0.252445 \\ 0. & 0. & 0. & 14.1561 & -5.40444 \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & 6.93889\times 10^{-18} & 1.05256 \\ 0. & 1. & 0. & -4.44089\times 10^{-16} & -1.28983 \\ 0. & 0. & 1. & -1.11022\times 10^{-16} & -0.467304 \\ 0. & 0. & 0. & 1. & -0.381773 \end{pmatrix}$$

Извод:x1=1.05256,x2=-1.28983,x3=-0.467304,x4=-0.381773

Детерминантата на матрицата на системата

(Debug) In[94]:=

$$Z = \begin{pmatrix} 1 & 2 & 0 & -4 & 0 \\ -3.12 & 5.76 & -21 & 0 & -0.9 \\ 89 & 0 & 7.87 & 0 & 90 \\ 0 & -9.8 & 0 & 34 & -0.34 \end{pmatrix};$$

```
n = Length[Z];
det = 1;
For[
 (*цикъл по стъпките*)
 m = 1, m ≤ n, m++,
 det = det * Z[[m, m]];
 Z[[m]] = Z[[m]] / Z[[m, m]]; (*първи етап*)
 For[s = 1, s ≤ n, s++,
  If[s ≠ m, Z[[s]] = Z[[m]] * (-Z[[s, m]]) + Z[[s]]]
 ]; (*втори етап*)
 Print[Z // MatrixForm]
]
Print["Det = ", det]
```

$$\begin{pmatrix} 1 & 2 & 0 & -4 & 0 \\ 0. & 12. & -21. & -12.48 & -0.9 \\ 0 & -178 & 7.87 & 356 & 90 \\ 0 & -9.8 & 0 & 34 & -0.34 \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 3.5 & -1.92 & 0.15 \\ 0. & 1. & -1.75 & -1.04 & -0.075 \\ 0. & 0. & -303.63 & 170.88 & 76.65 \\ 0. & 0. & -17.15 & 23.808 & -1.075 \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & 0.0497658 & 1.03356 \\ 0. & 1. & 0. & -2.02488 & -0.516779 \\ 0. & 0. & 1. & -0.56279 & -0.252445 \\ 0. & 0. & 0. & 14.1561 & -5.40444 \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & 6.93889\times10^{-18} & 1.05256 \\ 0. & 1. & 0. & -4.44089\times10^{-16} & -1.28983 \\ 0. & 0. & 1. & -1.11022\times10^{-16} & -0.467304 \\ 0. & 0. & 0. & 1. & -0.381773 \end{pmatrix}$$

Det = −51578.8

Намиране на обратната матрица

(Debug) In[99]:=

$$ZZ = \begin{pmatrix} 1 & 2 & 0 & -4 \\ -3.12 & 5.76 & -21 & 0 \\ 89 & 0 & 7.87 & 0 \\ 0 & -9.8 & 0 & 34 \end{pmatrix}; BB = \{0, -0.9, 90, -0.34\};$$

(Debug) In[100]:=

$$Z = \begin{pmatrix} 1 & 2 & 0 & -4 & 0 & 1 & 0 & 0 & 0 \\ -3.12 & 5.76 & -21 & 0 & -0.9 & 0 & 1 & 0 & 0 \\ 89 & 0 & 7.87 & 0 & 90 & 0 & 0 & 1 & 0 \\ 0 & -9.8 & 0 & 34 & -0.34 & 0 & 0 & 0 & 1 \end{pmatrix};$$

```
n = Length[Z];
det = 1;
For[
 (*цикъл по стъпките*)
 m = 1, m ≤ n, m++,
 det *= Z[[m, m]];
 Z[[m]] = Z[[m]] / Z[[m, m]]; (*първи етап*)
 For[s = 1, s ≤ n, s++,
  If[s ≠ m, Z[[s]] += Z[[m]] * (-Z[[s, m]])]
 ]; (*втори етап*)
 Print[Z // MatrixForm]
]
Print["Det = ", det]
Print["Пресметнато с WM det = ", Det[ZZ]]
Print["Пресметнато с WM solution = ", LinearSolve[ZZ, BB] // MatrixForm]
Print["Пресметнато с WM inverse matrix = ", Inverse[ZZ] // MatrixForm]
```

$$\begin{pmatrix} 1 & 2 & 0 & -4 & 0 & 1 & 0 & 0 & 0 \\ 0. & 12. & -21. & -12.48 & -0.9 & 3.12 & 1. & 0. & 0. \\ 0 & -178 & 7.87 & 356 & 90 & -89 & 0 & 1 & 0 \\ 0 & -9.8 & 0 & 34 & -0.34 & 0 & 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 3.5 & -1.92 & 0.15 & 0.48 & -0.166667 & 0. & 0. \\ 0. & 1. & -1.75 & -1.04 & -0.075 & 0.26 & 0.0833333 & 0. & 0. \\ 0. & 0. & -303.63 & 170.88 & 76.65 & -42.72 & 14.8333 & 1. & 0. \\ 0. & 0. & -17.15 & 23.808 & -1.075 & 2.548 & 0.816667 & 0. & 1. \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & 0.0497658 & 1.03356 & -0.0124415 & 0.00431995 & 0.0115272 & 0. \\ 0. & 1. & 0. & -2.02488 & -0.516779 & 0.506221 & -0.00215998 & -0.00576359 & 0. \\ 0. & 0. & 1. & -0.56279 & -0.252445 & 0.140698 & -0.0488533 & -0.00329348 & 0. \\ 0. & 0. & 0. & 14.1561 & -5.40444 & 4.96096 & -0.0211678 & -0.0564832 & 1. \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & 6.93889\times10^{-18} & 1.05256 & -0.0298817 & 0.00439437 & 0.0117258 & -0.00351549 \\ 0. & 1. & 0. & -4.44089\times10^{-16} & -1.28983 & 1.21583 & -0.00518779 & -0.0138429 & 0.143039 \\ 0. & 0. & 1. & -1.11022\times10^{-16} & -0.467304 & 0.337925 & -0.0496949 & -0.00553902 & 0.0397559 \\ 0. & 0. & 0. & 1. & -0.381773 & 0.350446 & -0.00149531 & -0.00399001 & 0.0706407 \end{pmatrix}$$

Det = -51578.8

Пресметнато с WM det = -51578.8

$$\text{Пресметнато с WM solution} = \begin{pmatrix} 1.05256 \\ -1.28983 \\ -0.467304 \\ -0.381773 \end{pmatrix}$$

$$\text{Пресметнато с WM inverse matrix} = \begin{pmatrix} -0.0298817 & 0.00439437 & 0.0117258 & -0.00351549 \\ 1.21583 & -0.00518779 & -0.0138429 & 0.143039 \\ 0.337925 & -0.0496949 & -0.00553902 & 0.0397559 \\ 0.350446 & -0.00149531 & -0.00399001 & 0.0706407 \end{pmatrix}$$

Зад3.Да се реши системата:

$5 + 67\,p - 4\,q = 32\,r$

r – 3.4 p + 34 = q

p + q + r = 1

Може да напишем системата и в този вид:

67p-4q-32r=-5

-3.4p-q+r=-34

p+q+r=1

(Debug) In[108]:=

$$R = \begin{pmatrix} 67 & -4 & -32 & -5 \\ -3.4 & -1 & 1 & -34 \\ 1 & 1 & 1 & 1 \end{pmatrix}$$

(Debug) Out[108]=

```
{{67, -4, -32, -5}, {-3.4, -1, 1, -34}, {1, 1, 1, 1}}
```

(Debug) In[109]:=

```
n = Length[R];
For[
 (*цикъл по стъпките*)
 m = 1, m ≤ n, m++,
 R[[m]] = R[[m]] / R[[m, m]]; (*първи етап*)
 For[s = 1, s ≤ n, s++,
  If[s ≠ m, R[[s]] = R[[m]] * (-R[[s, m]]) + R[[s]]]
 ]; (*втори етап*)
 Print[R // MatrixForm]
]
```

$$\begin{pmatrix} 1 & -\frac{4}{67} & -\frac{32}{67} & -\frac{5}{67} \\ 0. & -1.20299 & -0.623881 & -34.2537 \\ 0 & \frac{71}{67} & \frac{99}{67} & \frac{72}{67} \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & -0.44665 & 1.62531 \\ 0. & 1. & 0.51861 & 28.4739 \\ 0. & 0. & 0.92804 & -29.0993 \end{pmatrix}$$

$$\begin{pmatrix} 1. & 0. & 0. & -12.3797 \\ 0. & 1. & 0. & 44.7353 \\ 0. & 0. & 1. & -31.3556 \end{pmatrix}$$

Отговор:

p=-12.3797

q=44.7353

r=-31.3556

Общо решение: X=({{-12.3797}, {44.7353}, {-31.3556}})

Зад4. A*x=b→ $\begin{pmatrix} 2 & -1 & 0 & 3 \\ 4 & -2 & 5 & 0 \\ 3 & 5 & 2 & -3 \\ 0 & -2 & 1 & -1 \end{pmatrix}$ *x= $\begin{pmatrix} 9 \\ -10 \\ 0 \\ -7 \end{pmatrix}$

Решаваме по метода на Гаус-Жордан

(Debug) In[111]:=

$$K = \begin{pmatrix} 2 & -1 & 0 & 3 & 9 \\ 4 & -2 & 5 & 0 & -10 \\ 3 & 5 & 2 & -3 & 0 \\ 0 & -2 & 1 & -1 & -7 \end{pmatrix}$$

(Debug) Out[111]=

```
{{2, -1, 0, 3, 9}, {4, -2, 5, 0, -10}, {3, 5, 2, -3, 0}, {0, -2, 1, -1, -7}}
```

(Debug) In[112]:=

```
n = Length[K];
For[
 (*цикъл по стъпките*)
 m = 1, m ≤ n, m++,
 K[[m]] = K[[m]] / K[[m, m]]; (*първи етап*)
 For[s = 1, s ≤ n, s++,
  If[s ≠ m, K[[s]] = K[[m]] * (-K[[s, m]]) + K[[s]]]
 ]; (*втори етап*)
 Print[K // MatrixForm]
]
```

$$\begin{pmatrix} 1 & -\frac{1}{2} & 0 & \frac{3}{2} & \frac{9}{2} \\ 0 & 0 & 5 & -6 & -28 \\ 0 & \frac{13}{2} & 2 & -\frac{15}{2} & -\frac{27}{2} \\ 0 & -2 & 1 & -1 & -7 \end{pmatrix}$$

Power: Infinite expression $\frac{1}{0}$ encountered.

Infinity: Indeterminate expression 0 ComplexInfinity encountered.

Infinity: Indeterminate expression 0 ComplexInfinity encountered.

$$\begin{pmatrix} \text{Indeterminate} & \text{Indeterminate} & \text{ComplexInfinity} & \text{ComplexInfinity} & \text{ComplexInfinity} \\ \text{Indeterminate} & \text{Indeterminate} & \text{ComplexInfinity} & \text{ComplexInfinity} & \text{ComplexInfinity} \\ \text{Indeterminate} & \text{Indeterminate} & \text{ComplexInfinity} & \text{ComplexInfinity} & \text{ComplexInfinity} \\ \text{Indeterminate} & \text{Indeterminate} & \text{ComplexInfinity} & \text{ComplexInfinity} & \text{ComplexInfinity} \end{pmatrix}$$

Infinity: Indeterminate expression 0 ComplexInfinity encountered.

General: Further output of Infinity::indet will be suppressed during this calculation.

$$\begin{pmatrix} \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} \\ \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} \\ \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} \\ \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} \end{pmatrix}$$

$$\begin{pmatrix} \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} \\ \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} \\ \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} \\ \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} & \text{Indeterminate} \end{pmatrix}$$